OPi Primes: An Object–Intrinsic Theory of Primality

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Abstract

We present a rigorous axiomatic framework for OPi primes, a notion of primality intrinsic to mathematical objects rather than integers. Primes emerge as spectral obstructions detected via canonical logarithmic–modular transforms. This theory is developed axiomatically and applied to a variety of fundamental and special functions, including polynomials, trigonometric functions, the Gamma function, Bessel functions, and the Riemann zeta function. Classical primes appear as a special reduction of this framework.

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1 Axiomatization of OPi Primes

Axiom 1 (Object–Intrinsic Primality)**.** *Primality is an intrinsic invariant of a mathematical object $\mathcal{M}$ and is not defined a priori on the integers. Each object possesses its own prime spectrum.*

Axiom 2 (Existence of OPi Transform)**.** *For every admissible mathematical object $\mathcal{M}$, there exists an injective transform*

$$\mathcal{T}_{\mathcal{M}} : \mathbb{R}^+ \to \mathbb{C},$$

analytic except at isolated singularities determined solely by the internal structure of $\mathcal{M}$.

Definition 1 (OPi Phase Functional)**.** *The OPi phase associated with $\mathcal{M}$ is defined by*

$$\theta_{\mathcal{M}}(x) := \arg\big(\mathcal{T}_{\mathcal{M}}(x)\big),$$

with branch selection such that $\theta_{\mathcal{M}}$ is piecewise C^1.

Definition 2 (OPi Prime)**.** *A positive integer n is an OPi prime of $\mathcal{M}$ if*

$$\Delta\theta_{\mathcal{M}}(n) := \lim_{\varepsilon\to 0^+}\big(\theta_{\mathcal{M}}(n+\varepsilon) - \theta_{\mathcal{M}}(n-\varepsilon)\big) \neq 0.$$

Axiom 3 (Obstruction Equivalence)**.** *A nonzero phase jump at $x = n$ occurs if and only if $\mathcal{T}_{\mathcal{M}}$ encounters a structural obstruction at $x = n$, including but not limited to logarithmic singularities, poles, zeros, or branch points.*

Axiom 4 (Functoriality of OPi Spectra)**.** *If a morphism $F : \mathcal{M}_1 \to \mathcal{M}_2$ preserves analytic obstruction structure, then*

$$\mathrm{Spec}_{\mathrm{OPi}}(\mathcal{M}_1) \longrightarrow \mathrm{Spec}_{\mathrm{OPi}}(\mathcal{M}_2).$$

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2 OPi Primes of Polynomial Objects

Let

$$P(x) = \sum_{k=0}^{d} a_k x^k$$

be a nonconstant polynomial with complex roots $\{\rho_j\}_{j=1}^{d}$.

Definition 3 (Polynomial OPi Transform)**.** *The canonical OPi transform associated with P is*

$$\mathcal{T}_P(x) = \exp\left(i\pi\, x \log x \cdot \frac{P'(x)}{P(x)}\right).$$

Using partial fraction decomposition,

$$\frac{P'(x)}{P(x)} = \sum_{j=1}^{d} \frac{1}{x - \rho_j}.$$

Consequently, the OPi phase satisfies

$$\theta_P(x) = \pi x \log x \sum_{j=1}^{d} \Re\left(\frac{1}{x - \rho_j}\right).$$

Theorem 1 (Polynomial OPi Prime Criterion)**.** *An integer $n \in \mathbb{N}$ is an OPi prime of P if and only if there exists a root ρ_j such that*

$$\lim_{\varepsilon \to 0^+} \Re\left(\frac{1}{n + \varepsilon - \rho_j} - \frac{1}{n - \varepsilon - \rho_j}\right) \neq 0.$$

Equivalently,

$$\mathrm{Spec}_{\mathrm{OPi}}(P) = \{n \in \mathbb{N} \mid n \text{ coincides with, or lies on a branch induced by, } \Re(\rho_j) \text{ for some } j\}.$$

Corollary 1 (Non-Gaussian Primality)**.** *Polynomial OPi primes arise solely from analytic root obstructions and do not rely on Gaussian integers, lattice factorization, or arithmetic divisibility.*